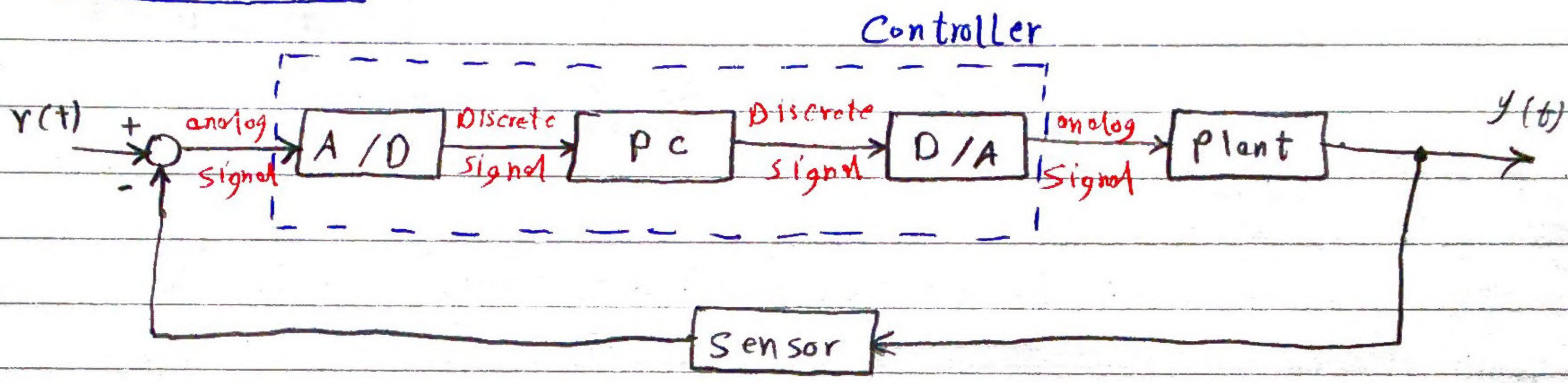
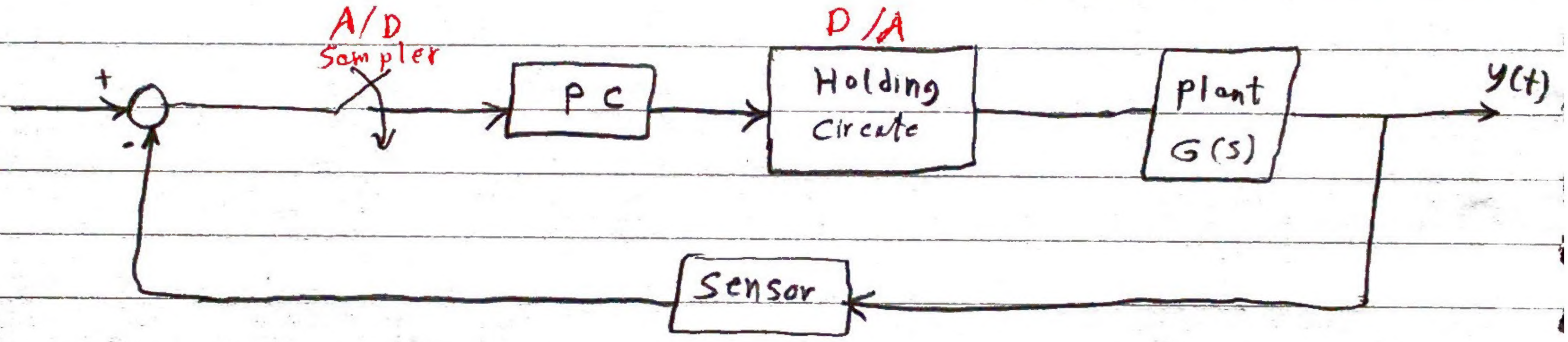
Lec 1

Revision on Z.T.

Digital system:





any Digital System Contains:

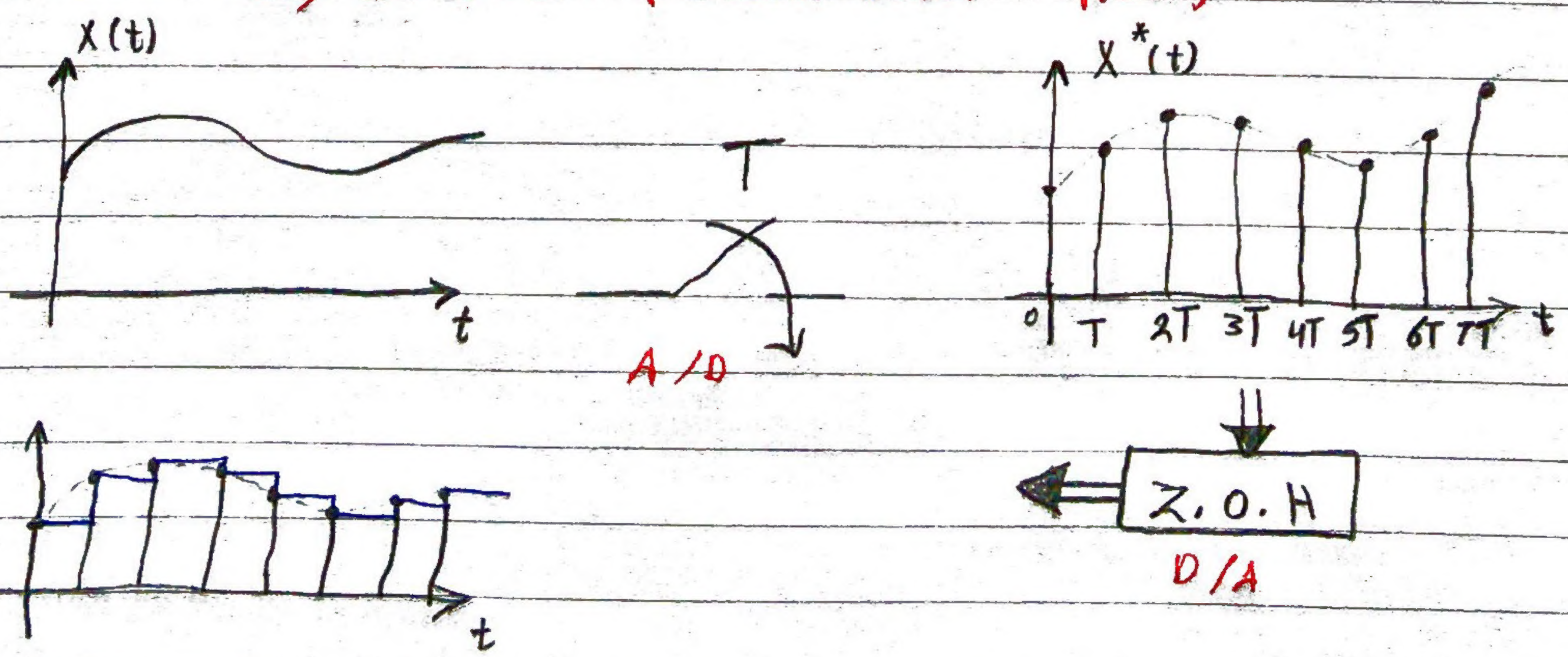
① A/D

② D/A Converter ≡ holding Circuite

→ Z.O.H (Zero order hold)

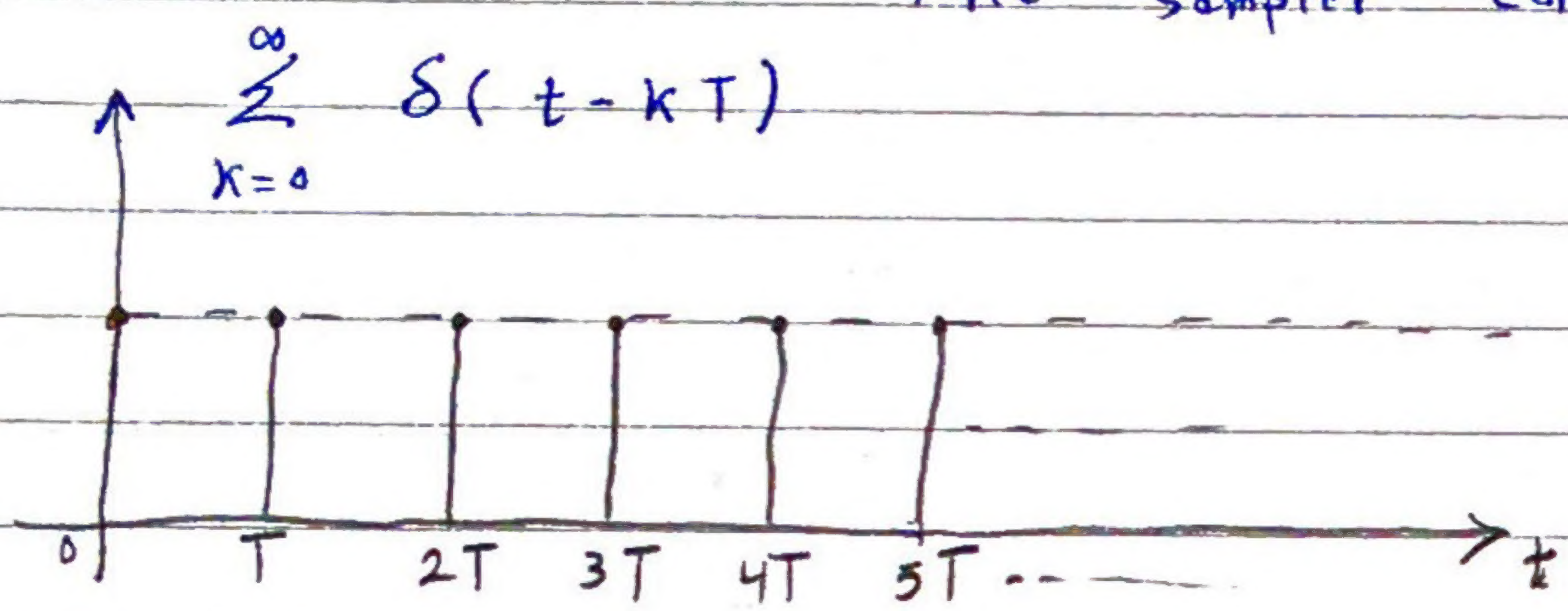
→ F.O.H (First order hold)

→ S.O.H (Second order hold)



Can be smothed using L.P.F

The sampler can be viewed



$$x^*(t) = \sum_{K=0}^{\infty} x(t)\delta(t-KT) = x(0) + x(T) + x(2T) + \cdots$$

$K \rightarrow$ Sample number

$T \rightarrow$ sample period

$x^*(t) \rightarrow$ output of the Sampler

$$x^*(t) = \sum_{K=0}^{\infty} x(t)\,\delta(t-KT) = \sum_{K=0}^{\infty} x(KT)\,\delta(t-KT)$$

$\rightarrow$ Laplace Transform

$$X^*(s) = \sum_{K=0}^{\infty} x(KT)\, e^{-KTS}$$

$\rightarrow$ Z. Trans Form $\quad z = e^{TS}$

$$X(z) = \sum_{K=0}^{\infty} x(KT)\, z^{-K}$$

$$x(t) \xrightarrow{Z.T.} X(z) = \sum_{K=0}^{\infty} x(KT)\, z^{-K}$$

Ex: $x(t) = u(t) = 1$

$$X(z) = \sum_{k=0}^{\infty} z^{-k} = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots$$

$$= \frac{1}{1 - z^{-1}} = \frac{z}{z-1}$$

Ex: $x(t) = e^{at}$

$$X(z) = \sum_{k=0}^{\infty} e^{akT} z^{-k}$$

$$= 1 + e^{aT} z^{-1} + e^{2aT} z^{-2} + \cdots$$

$$= \frac{1}{1 - e^{aT} z^{-1}} = \frac{z}{z - e^{aT}}$$

Ex: $x(t) = \delta(t)$

$$X(z) = \sum_{k=0}^{\infty} \delta(kT) z^{-k} = 1 + 0 + 0 + \cdots = 1$$

Ex: $x(t) = a^{t}$

$$X(z) = \sum_{k=0}^{\infty} a^{kT} z^{-k}$$

$$= 1 + a^{T} z^{-1} + a^{2T} z^{-2} + \cdots$$

$$= \frac{1}{1 - a^{T} z^{-1}} = \frac{z}{z - a^{T}}$$

Ex: $x(t) = t$

$$X(z) = \sum_{K=0}^{\infty} KT \, z^{-k}$$

$$= 0 + T z^{-1} + 2T z^{-2} + \cdots$$

$$z X(z) = T + 2T z^{-1} + 3T z^{-2} + \cdots$$

$$z X(z) - X(z) = T + T z^{-1} + T z^{-2} + \cdots$$

$$X(z)\,(z-1) = \frac{T}{1 - z^{-1}} = \frac{Tz}{z-1}$$

$$X(z) = \frac{Tz}{(z-1)^2}$$

Ex: $x(t) = \sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$

$$X(z) = \sum_{K=0}^{\infty} \sin(\omega KT)\, z^{-k}$$

$$= \sum_{K=0}^{\infty} \frac{1}{2j}\left[e^{j\omega KT} z^{-k} - e^{-j\omega KT} z^{-k}\right]$$

$$= \frac{1}{2j}\left[\left(1 + e^{j\omega T} z^{-1} + e^{2j\omega T} z^{-2} + \cdots\right) - \left(1 + e^{-j\omega T} z^{-1} + e^{-2j\omega T} z^{-2} + \cdots\right)\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1 - e^{j\omega T} z^{-1}} - \frac{1}{1 - e^{-j\omega T} z^{-1}}\right]$$

$$= \frac{1}{2j} \cdot \frac{\cancel{1} - e^{-j\omega T} z^{-1} - \cancel{1} + e^{j\omega T} z^{-1}}{1 - e^{j\omega T} z^{-1} - e^{-j\omega T} z^{-1} + z^{-2}}$$

$$= \frac{1}{2j} \cdot \frac{z\left(e^{j\omega T} - e^{-j\omega T}\right)}{z^2 - z\left(e^{j\omega T} + e^{-j\omega T}\right) + 1}$$

$$X(z) = \frac{z\left(\frac{e^{j\omega T} - e^{-j\omega T}}{2j}\right)}{z^2 - 2z\left(\frac{e^{j\omega T} + e^{-j\omega T}}{2}\right) + 1}$$

$$= \frac{z \sin(\omega T)}{z^2 - 2z\cos(\omega T) + 1}$$

Ex: $x(t) = \cos \omega t$

$$X(z) = \frac{z(z - \cos(\omega T))}{z^2 - 2z\cos(\omega T) + 1}$$

Properties of Z.T. :

$\boxed{1}$ $f_1(t) \pm f_2(t) \xrightarrow{Z.T} F_1(z) \pm F_2(z)$

$\boxed{2}$ $a\,f(t) \xrightarrow{Z.T} a\,F(z)$

$\boxed{3}$ $e^{\pm a t} f(t) \xrightarrow{Z.T} F(z\, e^{\mp a T})$ // عكس الإشارة //

$\boxed{4}$ $a^t f(t) \xrightarrow{Z.T} F\left(\frac{z}{a^T}\right)$

$\boxed{5}$ $t\,f(t) \xrightarrow{Z.T} -Tz \frac{dF(z)}{dz}$

$\boxed{6}$ initial value

$$f(0) = \lim_{t \to 0} f(t) = \lim_{z \to \infty} F(z)$$

$\boxed{7}$ Final value

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{z \to 1} (z-1)\,F(z)$$

Ex: $y(k+2) + 3y(k+1) + 2y(k) = \delta(k)$

if $y(0) = 0$, $y(1) = -1$, Solve For $y(k)$

→ using Z.T.

$z^2 y(z) - z^2 \cancel{y}(0) - z\,y(1) + 3[z\,Y(z) - z\,\cancel{y}(0)]$

$+ 2\,Y(z) = 1$

$z^2 Y(z) + z + 3z\,Y(z) + 2\,Y(z) = 1$

$Y(z)\,[z^2 + 3z + 2] = 1 - z$

$$Y(z) = \frac{1-z}{z^2+3z+2} = \frac{1-z}{(z+1)(z+2)}$$

$$= \frac{A}{z+1} + \frac{B}{z+2}$$

$A = 2$

$B = -3$

$$Y(z) = \frac{2}{z+1} - \frac{3}{z+2} = 2z^{-1}\,\frac{z}{z+1} - 3z^{-1}\,\frac{z}{z+2}$$

→ Inverse Z.T.

$$y(k) = 2\,(-1)^{k-1}\,u(k-1) - 3\,(-2)^{k-1}\,u(k-1)$$

Ex 8 $F(z) = \dfrac{z(z+1)}{(z+2)(z+4)}$

$$= z\left[\frac{A}{z+2} + \frac{B}{z+4}\right]$$

$A = \dfrac{-1}{2}$ $\qquad B = +\dfrac{3}{2} = 1.5$

$$= \frac{-1}{2}\,\frac{z}{z+2} + 1.5\,\frac{z}{z+4}$$

$$F(k) = \frac{-1}{2}(-2)^k\,u(k) + 1.5\,(-4)^k\,u(k)$$

Report. $F(z) = \dfrac{z(z+1)}{(z+2)(z+4)}$ For $T = 0.5$ sec